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Markov Switching Model as an Example of Non-stationary Exchange Rate Model

1. Introduction

Switching regressions models describe dynamics of processes that are subject to discrete (either rapid or gradual) changes with time. In other words, it is possible to observe periods, during which the process are generated by various regimes. The switching model applications are based on the general assumption that the time series can be modeled with the use of stochastic processes defined as sequences of random variables with a known type of conditional distribution in each regime. The switching model class considers both the changes in the economic phenomena behavior through definition of a different dynamic structure of the model in each regime and the changes of the value of their distribution parameters due to switching into another regime. In the case of switching models it is assumed that both the changes control mechanism within the particular regime and the regime shift mechanism are random. Thus, the specific nature of the models is the result of: the observed economic process and unobserved control process.[1] The name *Markov switching model* (MSM*)* defines the category of switching models where the stochastic process that controls the regime shifts is the homogeneous Markov chain.

The first references to the switching model can be found in the paper by Goldfeld and Quandt (1973), in which the linear regression model with time-varying coefficients was analyzed. The extension of this model is Markov switching model proposed by Hamilton (1994), where the parameters subject to occasional discrete shifts and the regime follows an unobserved Markov process.

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[1] J. Stawicki while describing Markov switching models introduces the notion of a double stochastic process, in order to emphasize the existence of the unobserved state variable apart from the observed economic variable; see Stawicki (2004).

The purpose of this paper is the description of selected exchange rate behaviors in relation to PLN with the use of Markov switching models. The paper includes a theoretical part with the specification Markov switching model and the empirical part including description of exchange rate returns behaviour, estimation and verification results of PLN exchange rate models.

2. Properties of Exchange Rate Return Distributions

The theory of financial markets often instead of taking into account financial prices it focuses on rates of return on these prices,[2] and at the same time logarithmic rates of return are most frequently used (see Jajuga (2000)):

$$r_t = \ln P_t - \ln P_{t-1} = \ln(\frac{P_t}{P_{t-1}}) \tag{1}$$

where: r_t – rate of return in t period, for $t = 2, ..., n$,
P_t – price of a financial instrument in t period,
P_{t-1} – price of a financial instrument in t-1 period.

Over a longer period of time, exchange return rates are characterized by the following properties (discussed in Franses (2004), Włodarczyk (2004)):

- Occurrence of the volatility clustering phenomenon – both large and small changes of exchange rates occur in series. After a large volatility period, a period of smaller volatility occurs there.
- Exchange return rate distributions are leptokurtic. The probability of occurrence of untypical (very large or very small) exchange rate fluctuations is higher than in the case of normal distribution. In the literature, another term for this phenomenon is also used, i.e. occurrence of "thick tails" in the rates of return of exchange rates distribution.
- These distributions are in many cases skew, which means that the rate of return distribution is not symmetrical around the average.
- Exchange rate fluctuations are negatively correlated with the changeability of their variance. Process variance depends on the previous rates of return, so when the exchange rate drops, there is a tendency towards an increase in the rates of return. This dependence is known as the leverage effect.
- There is a relationship between the variance of the exchange return rate and autocorrelation. The autocorrelation usually accompanies a small exchange rate variance and large volatility results in the lack of autocorrelation.
- Long-run data dependence, which means that after significant increases there are further increases, after which sudden decreases occur, followed by further decreases.

[2] Rates of return may be compared for various financial instruments. Moreover, prices, in contrast with rates of return, are usually non-stationary processes, which considerably hinders dynamic modeling of time series.

Such setting of exchange rates is a result of the nature of the processes which take place in the currency markets. Some of these properties are shown on the fig. 1 and 2.

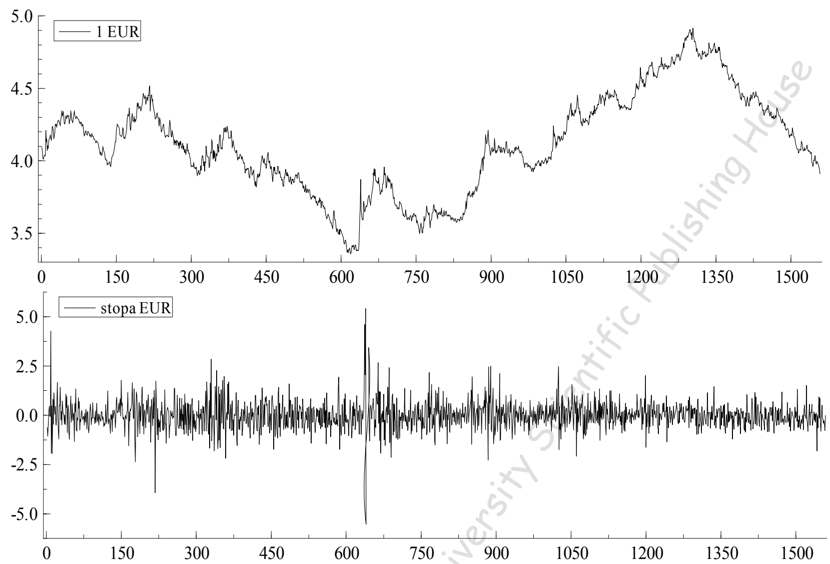

Fig. 1. Daily PLN/EUR exchange rate (top panel) and their logarithmic returns (bottom panel) in period 01.01.1999 –28.02.2005

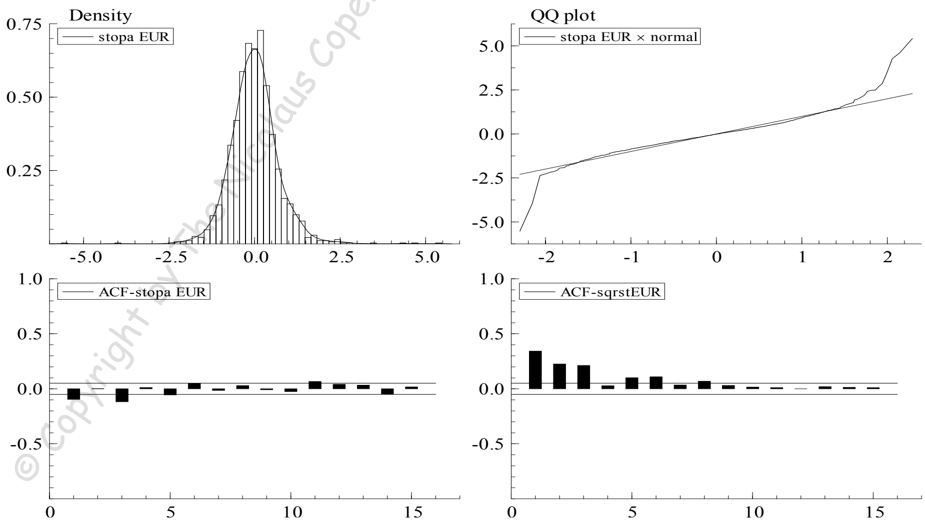

Fig. 2. Empirical distribution and QQ plot of PLN/EUR exchange rate returns (top panel) and autocorrelation functions for PLN/EUR returns and squared returns (bottom panel).

Exchange rates volatility depends on expected new information, among others on the decisions of Rada Polityki Pieniężnej (the Financial Policy Council), messages about the macroeconomic situation (e.g. concerning the inflation level, interest rates, gross national product, etc.). Their announcement dates are usually known in advance, and the uncertainty that concerns them results in an increase of the rates of return variance. This situation has a direct influence on the demand and supply of foreign exchange and currency and results in an increased exchange rate volatility.

Such behavior of exchange rates requires the use of special methods that would take account into the dependencies discussed above. There are various approaches in the literature for volatility modeling that try to capture these properties. Authors of this paper presents a proposal concerning the use of Markov model for detecting and describing regularities that govern the process of exchange rate fluctuation.

3. Heteroscedastic Markov Switching Model

A particular case of processes generated by Markov switching model are mixtures of distributions (*i.i.d. mixture distributions)*, in which the observed change in a variable is assumed to be random drawn from one of several distributions. An unobserved state variable s_t, that takes on the value in {1, 2, ..., N}, characterizes the state that the process was in at the moment *t*. The considerations concern the heteroscedastic specification of the model, i.e. the variance process is subject to random switches into various regimes with different volatility levels. It is worth to notice, that both the expected value of variable r_t and the variance of the innovation ε_t are functions of the state s_t:

$$r_t = \mu_{S_t} + \varepsilon_t, \ \varepsilon_t \sim N(0, \sigma_{s_t}^2) \tag{2}$$

where:

– s_t follows a first-order homogeneous Markov chain with N states and transition matrix P:

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & \vdots & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix}, \ p_{ij} \geq 0, \ \sum_{j=1}^{N} p_{ij} = 1, \tag{3}$$

for $i, j \in \{1,2,..,N\}$.

– Φ_t contains all the available information on the observable r_t process until the moment *t*.

The conditional distribution of the state variable s_t is the result of the division the conditional joint density- distribution of r_t and s_t by the unconditional density function[3] of r_t :

$$P(s_t = j \mid r_t, \Phi_{t-1}; \theta) = \frac{f(r_t, s_t = j \mid \Phi_{t-1}; \theta)}{f(r_t \mid \Phi_{t-1}; \theta)} =$$

$$= \frac{f(r_t \mid s_t = j, \Phi_{t-1}; \theta) \cdot P(s_t = j \mid \Phi_{t-1}; \theta)}{\sum_{i=1}^{N} f(r_t \mid s_t = i, \Phi_{t-1}; \theta) \cdot P(s_t = i \mid \Phi_{t-1}; \theta)} \tag{4}$$

for j = 1, 2, ..., N.

On the basis of the information set Φ_t , one may indicate with probability defined by the above estimation that the observation of the moment *t* was generated in an unobserved regime *j*.

One of the methods of the maximization the likelihood function of Markov switching model (2) is the application of the *Expectations Maximization* algorithm, originally proposed by Dempster, Laird, Rubin (1977). The application of the EM to the switching model is based on dependencies between the filter probabilities estimates (4) and the conditions imposed on the parameters through a system of equations resulting from the need to maximize the likelihood function. After fixing the initial values for all the model parameters, in each iteration of the EM algorithm two steps are made. The first step (*expectations)* consists in the determination of filter probabilities. The second step (*maximization)* leads to the determination of the parameter vector maximizing the likelihood function. Hamilton (1990) proved that the sequence of estimations so obtained is convergent with the local maximum likelihood function[4].

4. Markov Switching Model for PLN Exchange Rates

The heteroscedastic Markov switching model in the form (2) is the object of research, where the s_t variable means a two-state Markov chain: $s_t = 1$ for the regime of high exchange rate volatility, while $s_t = 2$ for the regime of low exchange rate volatility. Thus, Markov switching model provides information on the average levels of exchange return rates in each regime (the μ_1 and μ_2 parameters), the innovation variances characterizing the particular states (the σ_1^2 and σ_2^2 parameters) and the probabilities of remaining in the particular states (the p_{11} and p_{22} parameters).

[3] Depending on the Φ_t set of information range, two types of probabilities defined by the relationship (4) may be distinguished: *filter probability*, if the set Φ_t includes information available at the time t < T; *smoothed probability*, if set Φ_t includes the full sample of *ex post* available information for t = T.

[4] See: Hamilton (1990).

The empirical research were based on the average daily NBP quotations of PLN/USD exchange rate and PLN /CHF exchange rate between 02.01.1998 – 28.02.2005 and PLN/EUR exchange rate between 01.01.1999 – 28.02.2005. The parameter estimation of this model was made in the Ox package using the program codes written by Hamilton[5]:

Table 1. Parameters of Markov Switching Model – estimation result

Currency vector θ	PLN /EUR n = 1558	PLN /CHF n = 1813	PLN /USD n = 1813
μ_1	0.173334 (0.0937124)	0.248328 (0.0831591)	0.108561 (0.054556)
μ_2	-0.033976 (0.015884)	-0.0377841 (0.0162546)	-0.057882 (0.017087)
p_{11}	0.896028 (0.0342493)	0.88785 (0.0250588)	0.911902 (0.0247608)
p_{22}	0.981794 (0.00577913)	0.981469 (0.00491669)	0.964546 (0.010212)
σ_1^2	1.71207 (0.247454)	2.08652 (0.194277)	1.11068 (0.137022)
σ_2^2	0.290798 (0.0161122)	0.332519 (0.0152418)	0.248586 (0.0236443)
ρ_1	0.1490121	0.1418033	0.286956
$P(s_1 = 1/\Phi_T;\theta)$	0.0942	0.0145	0.0345
$P(s_T = 1/\Phi_T;\theta)$	0.0169	0.0237	0.0837
Log lik	- 99.4226	- 239.662	- 145.349

Source: Own calculations. Standard errors in parentheses.

Analyzing the estimation results a conclusion may be drawn that the expected values and variances in high and low volatility regimes differ from one another for each currency exchange rate. For example, the results obtained for the PLN/USD indicate that in the high volatility regime the average daily return rate on investments in USD is ca. 0,11%, which corresponds to the PLN depreciation, while in the low volatility regime, PLN was becoming by 0,06% stronger per day. The variance corresponding to the first regime is more than four times higher than the variance characterizing the second regime, which justifies the selection of the switching model for the description of the PLN/USD exchange rate behavior. It should also be noted that the probabilities of remaining the states of high and low volatility are large, which reflects the effect of volatility clustering in the series of return rates. The differences in parameter estimations for the particular currencies may result from their specific values.

Then the tests verifying the specification of the switching model were applied (cf. Hamilton (1996)).

The test results included in table 2 confirm that the residuals of Markov switching model for the PLN/USD exchange rate satisfy the white noise assumptions, i.e. are not serially correlated and are homoscedastic, both within each regime and across regimes. The LM test indicates the significant autocorrelation across regimes in case of PLN/EUR and PLN/CHF exchange rates. Markov property is satisfied for each currency rate.

[5] Authors are indebted to James Hamilton for sharing his programming codes.

Table 2. Diagnostics for dynamic specification of Markov switching model

Test	PLN /USD	PLN/EUR	PLN /CHF
White Test for Autocorrelation	10.3457	10.535346	9.684355
White Test for ARCH	9.529311	11.418597	12.974499
White Test of Markov Specification	3.91021	9.374302	11.970283
LM Test for Autocorrelation in Regime 1	0.623541	2.818197	4.727319
LM Test for Autocorrelation in Regime 2	0.025730	8.02875**	4.5622
LM Test for Autocorrelation Across Regimes	0.093286	10.105234**	7.655025**
LM Test for ARCH	4.20014	3.767907	5.622368

Source: Own calculations, * * is significant at the 1 % level.

An additional product of the estimation of Markov switching model parameters with the use of the EM algorithm are the filter and smoothed probabilities. Basing on them one may indicate with a defined probability that the return rate on investment in a foreign currency at the moment *t* was generated in the unobserved high or low volatility regime. Fig. 3 plots the estimated smoothed probabilities of being in the high volatility regime, as defined in (4):

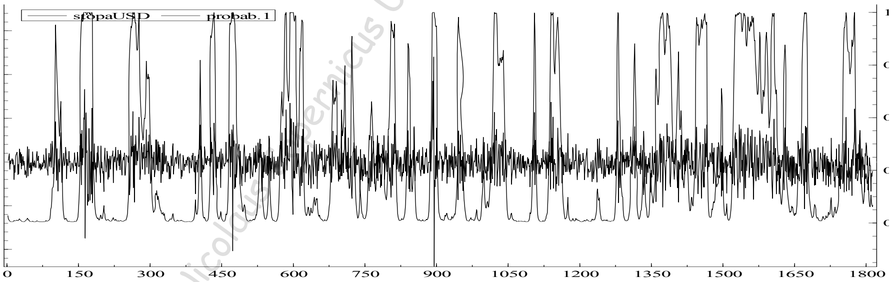

Fig. 3. The logarithmic return rates of PLN/USD exchange rate (left scale) against the smoothed probabilities of being in regime 1 (right scale).

One may also estimate the moment of switching the process from high volatility regime to that of low volatility (the following dependency must be satisfied: $P(s_t = 1/\Phi_T;\theta) > 0,5$ and $P(s_{t+1} = 1/\Phi_T;\theta) < 0,5$) and vice versa. For the series of PLN/USD exchange return rates, covering 1813 observations, 64 points of probable switching the process between the regimes of high and low volatility were distinguished. Analyzing fig.1, one should notice that the switches between regimes were not distributed evenly in time, so it confirms the occurrence of the volatility clustering in series of exchange return rate. For example, 6 switches took place in the period between 26.05.1998 and 15.09.1998, while in the period between 05.07.2001 and 24.07.2001 3 switches were observed and between 08.06.2004 and 14.06.2004 there were 2 switches. Among the factors that could have had an impact on the behavior of the PLN/USD rate in the mentioned months of 1998, the decisions of Rada Polityki Pieniężnej /Monetary Policy Council/ aimed at matching the currency rate policy to the direct inflationary target strategy, as well as making the exchange rate system more flexible, which enabled NBP to decrease the intervention scale in the currency market (as it appeared that the application of a wide range of oscillation band in case of Poland is a factor inducing a stabilizing speculation) and the Russian crisis. The Argentine crisis and the bad situation in the state finance explain the changes of regime in July, 2001. Within a few dozen of minutes, the dollar reinforced its position from 4,17 to 4,30 and the depreciation extent of the Polish zloty was comparable to the depreciation from the time of Polish flood or the Russian crisis.[6]

Markov switching models may describe various types of exchange rate behaviors (cf. Engle, Hamilton (1990)):

- asymmetry in the persistence of the particular regimes; the exchange rate increase may be short-term, but sharp (domestic currency depreciation phase: the μ_1 parameter reaches a large and positive value, p_{11} – a low value), while the decrease of exchange rate is gradual and long lasting (domestic currency appreciation phase: the μ_2 parameter reaches a negative and low by module value, p_{22} – a high value);
- the changes of the currency rate in a given period may be completely independent of the regime generating the process values in the preceding period, like in the random walk process ($p_{11} = 1 - p_{22}$);
- the process is characterized by sustained periods of domestic currency appreciation followed by sustained periods of its depreciation. This property of time series is called *long swings* in the exchange rate (the μ_1 and μ_2 have opposite signs and both p_{11} and p_{22} are high value).

The results of estimating the switching model parameters for PLN/USD, PLN/EUR and PLN/CHF exchange rate confirm the occurrence of the *long swings* effect. The transition probability values are the starting point for determination of the average length of the exchange rate appreciation and depreciation times (cf. Marsh (2000)):

$$a = \frac{1}{1 - p_{22}} \tag{5}$$

where: a – the average length of the appreciation period

$$d = \frac{1}{1 - p_{11}} \tag{6}$$

where: d – the average length of the depreciation period.

[6] www.parkiet.com.pl (01.2005)

Table 3. The average length of the appreciation and depreciation periods (in days)

Currency	Appreciation	Depreciation
PLN/USD	28.20556214	11.35099548
PLN/EUR	54.92694716	9.61797407
PLN/CHF	53.96362851	8.916629514

Source: Own calculations.

The analysis of the information included in table 3 leads to the finding that the average longest periods of PLN appreciation correspond to the PLN/EUR and PLN/CHF rate while the longest period of PLN depreciation corresponds to the PLN/USD rate as PLN lost its value in relation to USD through the average time of 11 subsequent days. In case of all the exchange rates under consideration, the PLN depreciation periods were shorter than those of appreciation.

In the short run, it is often believed that the exchange rate fluctuations are caused by rational expectations of the investors.[7] In other words, the investment decisions of currency market participants can suddenly increase the demand for a given currency and lengthen or shorten the length of the appreciation and depreciation periods.

5. Summary

While modeling short-term exchange rate fluctuations, it is usually necessary to consider random conditions, i.e. to give up the fundamental exchange rate theories in favor of the probabilistic modeling. The main advantages of Markov switching models include:

- they belong to the non-linear class of models,
- as a Markov chain describes state changes, the process may switch suddenly between regimes,
- the possibility of wide range description of the process dynamics, starting from frequent and relatively moderate changes to end with occasional and sharp ones,
- the distribution of the observed variable in this model is a mixture of several normal distributions, which does not imply a relation where the distribution of such variable is normal.

It is worth to emphasize that the most important aspect for currency risk management is the ability of the Markov switching model to calculate k-period ahead forecast of exchange rate volatility. It is due to the fact that knowledge about future volatility of exchange rates is useful in currency risk management process, in establishing monetary policy, in controlling international trade and foreign cash flows. Also volatility forecast enters option pricing formulas derived from the Black – Scholes model and its various extensions. For hedging

[7] The rational expectations theory and its applications in finance was described by, Osińska (2000)

against risk, portfolio management, volatility estimates are crucial too. Nowadays, banks and trading houses have to set aside reserve capital of at least three times that of value – at – risk. (VaR is defined as the minimum expected loss with a 1- percent confidence level for a given time horizon (usually one or ten days). Volatility forecast is needed to obtain such VaR estimates.

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